Confinement and gluon propagator in Coulomb gauge QCD

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We consider the role of the Faddeev-Popov determinant in the Coulomb gauge on the confinement properties of the QCD vacuum. We show that the determinant is needed to regularize the otherwise divergent functional integrals near the Gribov horizon but still enables large field configurations to generate IR enhanced running coupling. The physical gluon propagator is found to be strongly suppressed in the IR consistent with expectations from lattice gauge calculations.

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I. INTRODUCTION

Quantitative understanding of confinement and more generally of the dynamics of gluons at low energies remains to be a major challenge in QCD. In the past few years lattice simulations and phenomenological studies have provided new insights into the nature of the low energy behavior of the gluon propagator and the role of gluons in forming the hadronic spectrum [1,2]. Since gluons can only participate in strong interactions, spectroscopy of hadrons with excited gluonic modes is of crucial importance for investigations of confinement. It has recently been shown that hybrid mesons with excited quark and gluon modes should have properties similar to ordinary hadronic resonances and thus gluonic excitations may appear in the meson spectrum [3–7]. Searches for exotic mesons have produced a few tantalizing candidates [8–13] and new experiments planned for JLab and GSI focusing on light and charm meson spectra, respectively, are expected to produce a map of gluonic excitations.

In this paper we address gluon propagation in the QCD vacuum. This investigation was prompted by recent lattice results, indicating that in both covariant and Coulomb gauges the low momentum behavior of the gluon propagator is significantly different from the non-interacting one [14–18]. In Landau gauge it can be parametrized as an analytical function with a singularity (pole or branch point) at $|p^2| \sim O(1\ \mathrm{GeV}^2)$ [19]; however, due to uncertainties in the analytical structure the physical interpretation of the singularity as the gluon mass may be misleading. Nevertheless this is in the direction expected for physical degrees of freedom, e.g. two transverse gluons in a physical gauge [20–23]; an IR suppressed gluon propagator could originate from a large mass thus describing propagation over short instances only. In the four-dimensional Euclidean formulation of covariant gauge QCD, however, the lack of IR enhancement in the gluon propagator simultaneously contradicts the naive expectation that the color confining force might be simply related to the gluon propagator. A popular, phenomenological approach to gluon (and quark) low energy dynamics is based on a truncation of the self-consistent set of Dyson-Schwinger equations. In many such approaches the gluon propagator plays a central role in providing the effective interaction between quarks; for example, it is used to generate dynamical chiral symmetry breaking [24]. A soft gluon propagator implies that confinement has to be described by other means. For example, it has been argued that in covariant gauges the Kugo-Ojima confinement criterion for the absence of colored non-singlets in the physical spectrum can be satisfied with a soft gluon propagator if the ghost propagator is enhanced [1,25–28]. We will show that this also seems to be the case in Coulomb gauge formulation.

In a covariant formulation one sacrifices positivity constraints and the Fock space representation to introduce additional (ghost) non-physical degrees of freedom. Alternatively, by relaxing the requirement of manifest Lorentz covariance it is possible to eliminate all non-physical components and study confinement and other low energy phenomena within the framework of quantum mechanical wave functions. Such an approach has obvious, important implications for quark-model-based phenomenology. Furthermore, at finite density it allows for well established, diagrammatic, many-body techniques to be used. A many-body approach has proven to be successful in treating a variety of low energy phenomena in QCD. For example, the random phase approximation, which is typically (e.g. for the electron gas) relevant at high densities, in QCD is also applicable at low densities and may result in a self-consistent realization of confinement [23,29,30]. Because of the long-range nature of the confining interaction, at low-densities quasiparticle excitations have infinite energy which eliminates colored states from the physical spectrum. The absence of color singlet states is a consistent criterion for confinement in the Coulomb gauge as recently demonstrated in lattice simulations [31]. Due to bare quark-antiquark pairs near the Fermi-Dirac surface, the quasiparticle vacuum breaks chiral symmetry and leads to a non-vanishing scalar quark density. The collective excitations of this quark-antiquark plasma correspond to the Goldstone bosons [32–37].

The picture described above relies on the existence of a long range, quark-quark interaction. Such an interaction is expected to arise from the Coulomb operator which in the Coulomb gauge Hamiltonian describes direct interactions between (color) charge densities. Unlike QED, where this interaction is simply determined by the distance between charge sources, in QCD it is a complicated function of the transverse gluon field and cannot be thought of as a simple potential, i.e. of the Cornell type [38]. The conjecture that the Coulomb operator is related to the confining interaction is based on the observation that it is positive definite and enlarges at the Gribov horizon. The Gribov horizon defines the boundary of the gluon field domain. A number of approximations have been developed to calculate the expectation value of the Coulomb operator and to verify this conjecture [23,29,39–41]. In the process it has been realized that the Gribov region still contains physically equivalent field configurations. To what extent the necessary identification of the wave functional at these gauge-equivalent points modifies the expectation value of the Coulomb operator remains an open issue [41].

It should also be noted that confinement of the Coulomb operator is not equivalent to confinement of the static potential. Furthermore, Coulomb confinement is a necessary but not sufficient condition for confinement of the static potential [27]. The relation between the two has recently being explored in Ref. [31]. For example, Coulomb confinement respects Casimir scaling while in the adjoint representation the static potential is expected to be screened.

The standard approach to a many-body system is to introduce a physically motivated ansatz for the ground state wave functional and to define approximations for the evaluation of expectation values. The variational mean field approach is expected to be particularly adequate for systems with long range correlations which is the case here due to the Coulomb operator. Furthermore, it can in principle be systematically improved by the introduction of two-, three-, and more particle correlations [37]. We have followed this approach in Ref. [23] where we generated the confining interaction from the Coulomb operator but a specific assumption on the behavior of the gluon propagator at low energies had to be imposed. In particular, the gluon dispersion relation which follows from minimizing the vacuum expectation value was solved self-consistently together with the confining interaction with a specific boundary condition imposed on the propagator. In absence of the Faddeev-Popov determinant this boundary condition was necessary in order to obtain a non-trivial solution to the coupled integral equations.

In this paper we will show how the Faddeev-Popov determinant constraints the low momentum behavior of the gluon propagator and brings it into a qualitative agreement with lattice results. The paper is organized as follows: in Sec. II we give a brief description of QCD in the Coulomb gauge. As discussed above, the main novel feature of this approach is the inclusion of the Faddeev-Popov determinant. The Faddeev-Popov determinant has been treated previously in the case of the S^3 spatial manifold with an UV cutoff which results in a finite number of normal modes of the gluon field [39,42]. This is quite different from the continuum, flat three-dimensional case studied here.

The details of the many-body formulation are given in Sec. III. Even though the main focus of this work is the IR sector, for completeness we briefly discuss the UV behavior and renormalization in Sec. IV. It should be stressed, however, that the main results of this work could well be obtained by simply cutting off the high momentum components. The numerical results are discussed in Sec. V.

II. QCD IN THE COULOMB GAUGE

The QCD Coulomb gauge Hamiltonian, defined by $\boldsymbol{\nabla}\cdot A^a(\mathbf{x})=0$ is given by [43]

$$H=H[\mathbf{A}^a(\mathbf{x}),\boldsymbol{\Pi}^a(\mathbf{x})]=H_0+H_{qg}+H_{g^3}+H_{g^4}+H_C, \quad (1)$$

with $\boldsymbol{\Pi}^a(\mathbf{x})$ being the canonical momentum, satisfying

$$[\Pi^a_i(\mathbf{x}),A^b_j(\mathbf{y})]=-i\,\delta_{ab}\delta^{ij}_T(\boldsymbol{\nabla})\,\delta^3(\mathbf{x}-\mathbf{y}), \quad (2)$$

$\delta^{ij}_T(\boldsymbol{\nabla})=\delta^{ij}-\nabla^i\nabla^j/\boldsymbol{\nabla}^2$ and in the Shrödinger representation given by $\boldsymbol{\Pi}^a(\mathbf{x})=-i\,\delta/\delta\mathbf{A}^a(\mathbf{x})$. The five terms represent the kinetic energy, the quark-transverse gluon coupling, the magnetic three- and four-gluon couplings and the instantaneous Coulomb energy, respectively. In this paper we focus on the gluon sector and thus will ignore quark degrees of freedom. The gluon kinetic term is given by

$$H_0=\frac{1}{2}\int d\mathbf{x}[\mathcal{J}^{-1}\boldsymbol{\Pi}^a(\mathbf{x})\mathcal{J}\boldsymbol{\Pi}^a(\mathbf{x})+(\boldsymbol{\nabla}\times\mathbf{A}^a(\mathbf{x}))^2], \quad (3)$$

with

$$\mathcal{J}\equiv \mathrm{Det}(1-\lambda)=e^{\mathrm{Tr}\log(1-\lambda)} \quad (4)$$

being the Faddeev-Popov (FP) determinant. The matrix λ is given by

$$\lambda_{\mathbf{x},a;\mathbf{y},b}=\left(-\frac{1}{\boldsymbol{\nabla}^2}\right)_{\mathbf{x},\mathbf{y}} g f_{acb}\mathbf{A}^c(\mathbf{y})\boldsymbol{\nabla}_{\mathbf{y}}, \quad (5)$$

and in Eq. (4) the trace is over the spatial $\mathbf{x},\mathbf{y}$ and color a,b,c indices. The FP determinant is the Jacobian of transformation from the canonical coordinates of the Weyl gauge, $\mathbf{V}^a(\mathbf{x}), V^{0,a}(\mathbf{x})=0$, with kinetic energy given by $1/2\int d\mathbf{x}[-i\,\delta/\delta\mathbf{V}^a(\mathbf{x})]^2$, to the Coulomb gauge coordinates $\mathbf{A}^a(\mathbf{x})$ defined through the gauge map

$$\mathbf{V}^a(\mathbf{x})=u(\vec{\phi})\mathbf{A}^a u^{-1}(\vec{\phi})+iu(\vec{\phi})\boldsymbol{\nabla}u^{-1}(\vec{\phi}), \quad (6)$$

with $\boldsymbol{\nabla}\cdot\mathbf{A}^a(\mathbf{x})=0$. The dependence of the Hamiltonian and wave functionals on the N_c^2-1 Euler angles $\vec{\phi}(\mathbf{x})$ can be eliminated using the Gauss's law constraint and results in the Coulomb energy term [43],

$$H_C=\frac{1}{2}\int d\mathbf{x}d\mathbf{y}\mathcal{J}^{-1}\rho^a(\mathbf{x})\mathcal{J}K[\mathbf{A}]_{\mathbf{x},a;\mathbf{y},b}\rho^b(\mathbf{y}), \quad (7)$$

with $\rho^a(\mathbf{x})$ being the color charge density, which in the absence of quarks is given by

$$\rho^a(\mathbf{x})=f_{abc}\boldsymbol{\Pi}^b(\mathbf{x})\mathbf{A}^c(\mathbf{x}), \quad (8)$$

and the Coulomb kernel K given by

$$K[\mathbf{A}]_{\mathbf{x},a;\mathbf{y},b}\equiv g^2\left[(1-\lambda)^{-2}\left(-\frac{1}{\boldsymbol{\nabla}^2}\right)\right]_{\mathbf{x},a;\mathbf{y},b}. \quad (9)$$

More details of the derivation of the Coulomb gauge can be found in Refs. [23,43].

Functional integrals in the Coulomb gauge are performed over the measure $\Pi_{\mathbf{x},a,i}dA^{i,a}(\mathbf{x})\mathcal{J}$. The Faddeev-Popov determinant results from the nonlinear field transformation given in Eq. (6) and reflects the complicated topology of the field space domain. Furthermore, it is well known that the

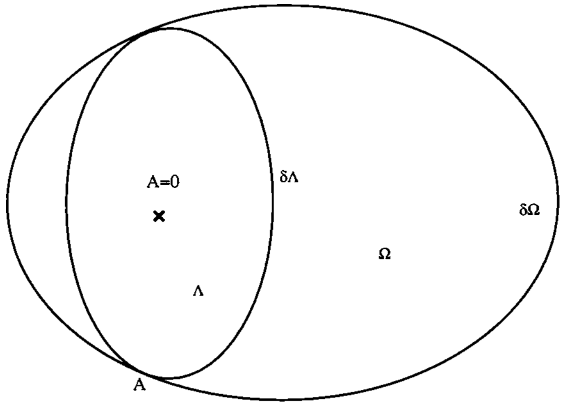

FIG. 1. A schematic representation of the field domain in the Coulomb gauge. The point A is on the common boundary of the fundamental modular region ($\delta\Lambda$) and the Gribov region ($\delta\Omega$) and corresponds to a coordinate singularity, $\mathcal{J}[A]=0$.

gauge condition, $\nabla\cdot\mathbf{A}^a=0$ is not a complete gauge fixing and thus the mapping $V\to A,\vec{\phi}$ is not unique. The unique solution on a gauge orbit can be defined as the absolute minimum of the functional, $I[A,g]=\int d\mathbf{x}[\mathbf{A}^a(\mathbf{x})]_g^2$ minimized over g [41]. At the minimum of $I[A]$, $\nabla\cdot\mathbf{A}^a=0$ and the FP operator is positive. The space of the absolute minima defines the fundamental modular region (FMR), Λ as shown in Fig. 1. The boundary of Λ is a set of gauge fields which lead to degenerate absolute minima. The fundamental region resides inside the so-called Gribov region Ω, corresponding to all minima of $I[A]$ and thus also satisfying the transversality condition. The boundary $\partial\Omega$ defines a set of configurations for which the gauge mapping is singular and $\mathcal{J}[A]=0$. In what follows we will primarily study the role of configurations near singular boundary points on $\partial\Omega$. Since there exist configurations for which $\partial\Lambda$ and $\partial\Omega$ overlap, the point $A^a(\mathbf{x})=0$ lies in Λ and both Λ and Ω are convex, some fluctuations around the null field reach the coordinate singularity, $\delta\Omega$ without leaving Λ. Furthermore, it has been pointed out [44] that it is the common boundary points which dominate functional integrals. The argument is based on the observation that in the infinite volume limit a probability distribution may be concentrated on a lower dimensional subspace, i.e. at the boundary. Thus even if field configurations outside of FMR are included these may not lead to substantial errors.

In the limit $\mathcal{J}=1$, the kinetic term describes a set of coupled harmonic oscillators and its (unnormalized) ground state $|\omega_0\rangle$ is given by

$$\langle\mathbf{A}|\omega_0\rangle=\exp\left(-\frac{1}{2}\int\frac{d\mathbf{k}}{(2\pi)^3}\,\omega_0(k)\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})\right) \tag{10}$$

with $\omega_0(k)=k=|\mathbf{k}|$ being the free gluon energy and

$$\mathbf{A}^a(\mathbf{k})\equiv\int d\mathbf{x}e^{-i\mathbf{k}\cdot\mathbf{x}}\mathbf{A}^a(\mathbf{x}) \tag{11}$$

represents the normal modes.

The determinant of the FP operator leads to a suppression of the ground state wave functional near the Gribov horizon. This can be illustrated using an analogy between the Weyl and the Coulomb gauge kinetic terms and a harmonic oscillator in Cartesian and spherical coordinates, respectively. In N dimensions, the S-wave harmonic oscillator radial wave function satisfies

$$\frac{1}{2}\sum_{i=1}^{N}\left[-\frac{\partial^2}{\partial x_i^2}+\omega^2x_i^2\right]R(r)=\frac{1}{2}\left[\frac{1}{\mathcal{J}}\frac{\partial}{\partial r}\mathcal{J}\frac{\partial}{\partial r}+\omega^2r^2\right]R(r). \tag{12}$$

Here the Jacobian is given by $\mathcal{J}=r^{N-1}\sim\exp(-N\log r)$ and it vanishes at the boundary $r\to0$ of the domain of r. The ground state wave function $R(r)$ is finite at that boundary, $R(r)=\exp(-r^2\omega^2/2)$ but the radial wave function defined by $u(r)=\mathcal{J}^{1/2}R(r)$ vanishes as $r\to0$. The Hamiltonian can be redefined to absorb the Jacobian,

$$H\to\bar{H}=\mathcal{J}^{1/2}H\mathcal{J}^{-1/2}=\frac{1}{2}(p_r^2+\omega^2r^2)+V_C, \tag{13}$$

where $p_r=-i\partial/\partial r$ and the additional potential is given by $V_C=\mathcal{J}^{-2}[p_r,\mathcal{J}]^2/4-\mathcal{J}^{-1}[p_r,[p_r,\mathcal{J}]]/2$. The Hamiltonian $\bar{H}$ is Hermitian with respect to a flat measure in the radial direction,

$$\frac{\int D\mathbf{x}R(\mathbf{x})HR(\mathbf{x})}{\int d\Omega}=\int dr\mathcal{J}R(r)HR(r)=\int dru(r)\bar{H}u(r). \tag{14}$$

In terms of $\bar{H}$ and $u(r)$ one effectively recovers the simple harmonic motion in one dimension (modulo V_C), except for the boundary condition $u(r)\to0$ at the point $r=0$ corresponding to the singularity of the coordinate transformation. Furthermore, by transforming to the radial basis $R\to u$ the integrals are to be performed over a flat measure. The correspondence with QCD goes as follows. In the Coulomb gauge, at the Gribov horizon, $\mathcal{J}=0$ as the Coulomb kernel diverges and this can be interpreted as a manifestation of confinement [23]. However, since the wave functional vanishes at singular points $\partial\Omega$ of the coordinate transformation (equivalent to $u\to0$ as $r\to0$), functional integrals over color singlet states are expected to be finite. In Ref. [23] we have explored this scenario, but we have not accounted for the boundary condition on the ground state wave functional. In effect we used a Gaussian ansatz for u instead of R, i.e. the radial wave functional was finite at the Gribov horizon. To ensure that functional integrals which include the FP operator or the Coulomb operator converge we had to choose a particular condition on the parameters of the ground state wave functional. Summarizing, the FP determinant is a crucial element of the QCD Coulomb gauge dynamics as it suppresses the integrands near the Gribov horizon. This suppression can be accounted for explicitly in the measure [i.e. the middle term in Eq. (14)] or absorbed into the radial wave function [the last term in Eq. (14)]. In the following we will keep $\mathcal{J}$ explicitly in the measure and use the Gaussian functional for the ground state (i.e. R).

As long as gauge fields are within the Gribov region Ω, the Coulomb potential is positive and it is possible to use a variational approach. The Gaussian ansatz can be motivated by noting that mean-field approximation works best for systems with long-range correlations and from a technical standpoint it enables diagrammatic expansion, which corresponds to generalized quasiparticle approximation. This approximation can be systematically improved using cluster expansion techniques and excited states can also be studied [37]. In this context one often uses the formalism of second quantization which is natural when dealing with Gaussian integrals over polynomials (Wick theorem). In our case, however, before one gets to this stage one has to deal with non-polynomial operators, e.g. $\mathcal{J}[A]$ or $K[A]$, and thus it is simpler to continue with the Schrödinger representation.

III. THE QUASIPARTICLE SPECTRUM

In the non-interacting case $H=H_0$ the perturbative vacuum of Eq. (10) minimizes the energy density of the system, i.e.

$$0=\frac{\partial}{\partial\omega(k)}\frac{\langle\omega|H_0|\omega\rangle/\mathcal{V}}{\langle\omega|\omega\rangle}\bigg|_{\omega=\omega_0} =\frac{\partial}{\partial\omega(k)}\frac{1}{4}\int\frac{d\mathbf{q}}{(2\pi)^3}\left[\omega(q)+\frac{\mathbf{q}^2}{\omega(q)}\right]_{\omega=\omega_0}. \quad (15)$$

Here $\mathcal{V}$ is the total number of gluon degrees of freedom, $\mathcal{V}=\delta_{aa}\delta_T^{ii}\int d\mathbf{x}=(N_C^2-1)\times 2\times$volume. To describe the quasiparticle spectrum we will use the same Gaussian variational ansatz. The VEV of the Hamiltonian becomes

$$E(\omega)=\langle\omega|H\omega\rangle/\langle\omega|\omega\rangle\equiv E_K(\omega)+E_C(\omega), \quad (16)$$

where

$$E_K=\frac{1}{2\langle\omega|\omega\rangle}\int D\mathbf{A}^a\int d\mathbf{x}e^{-\int d\mathbf{k}[\omega(k)/2]\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})} \times\{\mathbf{\Pi}^a(\mathbf{x})\mathcal{J}\mathbf{\Pi}^a(\mathbf{x})+[\boldsymbol{B}^a(x)]^2\}e^{-\int d\mathbf{k}[\omega(k)/2]\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})}, \quad (17)$$

$\mathbf{B}^a(\mathbf{x})=\boldsymbol{\nabla}\times\mathbf{A}^a(\mathbf{x})+gf_{abc}\mathbf{A}^b(x)\times\mathbf{A}^c(\mathbf{x})/2$, and

$$E_C=\frac{1}{2\langle\omega|\omega\rangle}\int D\mathbf{A}^a\int d\mathbf{x}d\mathbf{y}e^{-\int d\mathbf{k}[\omega(k)/2]\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})} \times[\rho^a(\mathbf{x})\mathcal{J}K[\mathbf{A}]_{\mathbf{x},a;\mathbf{y},b}\rho^b(\mathbf{y})]e^{-\int d\mathbf{k}[\omega(k)/2]\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})}. \quad (18)$$

The ground state normalization is given by

$$\langle\omega|\omega\rangle=\int D\mathbf{A}^a\mathcal{J}[A]e^{-\int d\mathbf{k}\omega(k)\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})}\equiv\langle\mathcal{J}[A]\rangle. \quad (19)$$

We will use $\langle\cdots\rangle$ to represent functional integrals over the ground state ansatz functional with the *flat* measure. After integrating by parts the kinetic and Coulomb kernel contributions can be written as $[[d\mathbf{k}]\equiv d\mathbf{k}/(2\pi)^3]$

$$E_K=\frac{1}{2\langle\omega|\omega\rangle}\int D\mathbf{A}^a\mathcal{J}e^{-\int d\mathbf{k}\omega(k)\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})} \times\left[\int[d\mathbf{k}]\omega^2(k)\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})+\int d\mathbf{x}[\boldsymbol{B}^a(x)]^2\right] \quad (20)$$

and

$$E_C=\frac{1}{2\langle\omega|\omega\rangle}\int D\mathbf{A}^a\int\Pi_i^4[d\mathbf{k}_i]\mathcal{J}e^{-\int d\mathbf{k}\omega(k)\mathbf{A}^a(\mathbf{k})\mathbf{A}^a(-\mathbf{k})} \times\rho^a(\mathbf{k}_1,\mathbf{k}_2)K[\mathbf{A}]_{\mathbf{k}_i,a,b}\rho^b(\mathbf{k}_3,\mathbf{k}_4), \quad (21)$$

respectively. The charge density is now given by

$$\rho^a(\mathbf{k}_i,\mathbf{k}_j)=f_{abc}\omega(k_j)\mathbf{A}^b(\mathbf{k}_i)\cdot\mathbf{A}^c(\mathbf{k}_j) \quad (22)$$

and the Coulomb kernel by

$$K[\mathbf{A}]_{\mathbf{k}_i,a,b}=\int d\mathbf{x}d\mathbf{y}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{x}}K[\mathbf{A}]_{\mathbf{x},a;\mathbf{y},b}e^{i(\mathbf{k}_3+\mathbf{k}_4)\cdot\mathbf{y}}. \quad (23)$$

Even though details of the boundary of the functional integrals are not known, the partial integration is presumably justified since the integrand vanishes as $\mathbf{A}\to\infty$ and at the boundary of the Gribov region, $\mathcal{J}\to 0$. Compared to the harmonic oscillator example discussed earlier, the partial integration combines contributions from V_C and p_r^2 in Eq. (13) to the vacuum expectation value (VEV) of the Hamiltonian and expresses the sum as a coordinate space integral over the Gaussian wave function. In our case the complication in evaluating functional integrals over $D\mathbf{A}$ is due to the nonlinear dependence of $\mathcal{J}[A]$ and $K[A]$ on the A through the FP operator, $(1-\lambda)=(1-\lambda[\mathbf{A}])$. These integrals are performed by expanding functionals in powers of A, performing Gaussian integrals of over polynomials in A and approximating them by products of two-point Green's functions. This will be illustrated in particular cases below.

To simplify the notation, the triplet of indices representing momentum, color and spin will be denoted by Greek letters, e.g. $\alpha=(\mathbf{k},a,i)$ and a doublet containing a momentum and a color index by $\overline{\alpha}=(\mathbf{k},a)$. The summation convention will be used with upper and lower indices differing by a replacement $\mathbf{k}\to-\mathbf{k}$, e.g. $A^\alpha\equiv A^{ia}(\mathbf{k})$,

$$\sum_\alpha A^\alpha A^\alpha\equiv A^\alpha A_\alpha=\sum_a\sum_{ij}\int[dk]A^{ia}(\mathbf{k})\delta_T^{ij}(\mathbf{k})A^{ja}(-\mathbf{k}). \quad (24)$$

In this notation the matrix λ in the FP operator can be written as

$$\lambda^{\bar{\alpha}}{}_{\bar{\beta}} = \lambda^{(\mathbf{p},a)}{}_{(\mathbf{q},b)} \equiv \lambda^{\bar{\alpha}}{}_{\gamma\bar{\beta}} A^{\gamma} = g f_{acb} \int [d\mathbf{k}] (2\pi)^3 \delta^3(\mathbf{p}-\mathbf{k}-\mathbf{q}) \frac{\mathbf{A}^c(\mathbf{k})\cdot i\mathbf{q}}{\mathbf{p}^2}, \quad (25)$$

with

$$\lambda^{\bar{\alpha}}{}_{\gamma\bar{\beta}} = \lambda^{(\mathbf{p},a)}{}_{(\mathbf{k},c,k),(\mathbf{q},b)} = (2\pi)^3 \delta^3(\mathbf{p}-\mathbf{k}-\mathbf{q}) \frac{ig[\delta_T(\mathbf{k})q]^k}{\mathbf{p}^2} f_{acb}. \quad (26)$$

Evaluation of functional integrals is simplified by introducing generating functionals,

$$\langle A^{\alpha_1}\cdots A^{\alpha_n} F[A]\rangle = \int \mathcal{D}A F[A] A^{\alpha_1}\cdots A^{\alpha_n} e^{-\sum_\gamma \omega_\gamma A^\gamma A_\gamma} \quad (27)$$

with $\mathcal{D}A = \Pi_\alpha dA^\alpha$ and since

$$\int \mathcal{D}A F[A] e^{\sum_\gamma [-\omega_\gamma A^\gamma A_\gamma + A^\gamma J_\gamma]} = e^{\sum_\gamma (J^\gamma J_\gamma/4\omega_\gamma)} \left\langle F\left[A + \frac{J}{2\omega}\right]\right\rangle \quad (28)$$

we obtain

$$\langle A^{\alpha_1}\cdots A^{\alpha_n} F[A]\rangle = \frac{\delta}{\delta J_{\alpha_1}}\cdots \frac{\delta}{\delta J_{\alpha_n}}\bigg|_{J=0} e^{\sum_\gamma (J^\gamma J_\gamma/4\omega_\gamma)} \left\langle F\left[A + \frac{J}{2\omega}\right]\right\rangle. \quad (29)$$

In this paper we are primarily concerned with the instantaneous part of the transverse gluon propagator (two-point Green's function) defined by

$$\Pi^\alpha{}_\beta \equiv \langle A^\alpha A_\beta \mathcal{J}[A]\rangle / \langle \mathcal{J}[A]\rangle = \frac{\delta^\alpha{}_\beta}{2\Omega_\alpha} \quad (30)$$

with the last equality following from translational invariance and color neutrality of the vacuum. In the approximation $\mathcal{J}=1$, one has $\Omega(k)=\omega(k)$ and one obtains the propagator used in Ref. [23]. From Eq. (29) it follows that

$$\Pi^\alpha{}_\beta = \frac{\delta}{\delta J_\alpha}\frac{\delta}{\delta J^\beta}\bigg|_{J=0} e^{\sum_\gamma (J^\gamma J_\gamma/4\omega_\gamma)} \left\langle \mathcal{J}\left[A+\frac{J}{2\omega}\right]\right\rangle \Big/ \langle \mathcal{J}[A]\rangle = \frac{1}{2\omega_\alpha}\left[\delta^\alpha{}_\beta + 2\omega_\alpha \frac{\delta}{\delta J_\alpha}\frac{\delta}{\delta J^\beta}\bigg|_{J=0} \left\langle \mathcal{J}\left[A+\frac{J}{2\omega}\right]\right\rangle \Big/ \langle \mathcal{J}[A]\rangle\right], \quad (31)$$

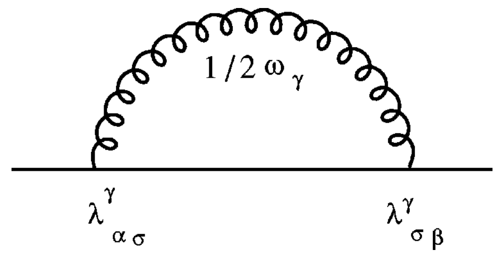

FIG. 2. Primitive self-energy, i.e. the lowest order correction to the FP operator, or the Coulomb line I^0. The solid line represents the bare Coulomb potential ($1/p^2$).

with the first term corresponding to the propagator in the absence of the FP determinant ($\mathcal{J}=1$) and the second term given by

$$\frac{\delta}{\delta J_\alpha}\frac{\delta}{\delta J^\beta}\bigg|_{J=0}\left\langle \mathcal{J}\left[A+\frac{J}{2\omega}\right]\right\rangle \Big/ \langle \mathcal{J}[A]\rangle = -\left\langle \left[\frac{\lambda^\alpha}{2\omega_\alpha}(1-\lambda)^{-1}\frac{\lambda_\beta}{2\omega_\beta}(1-\lambda)^{-1}\right]^{\bar{\gamma}}{}_{\bar{\gamma}} \mathcal{J}[A]\right\rangle \Big/ \langle \mathcal{J}[A]\rangle + \left\langle \left[\frac{\lambda^\alpha}{2\omega_\alpha}(1-\lambda)^{-1}\right]^{\bar{\gamma}}{}_{\bar{\gamma}} \left[\frac{\lambda_\beta}{2\omega_\beta}(1-\lambda)^{-1}\right]^{\bar{\sigma}}{}_{\bar{\sigma}} \mathcal{J}[A]\right\rangle \Big/ \langle \mathcal{J}[A]\rangle, \quad (32)$$

where $[\lambda^\alpha]^{\bar{\gamma}}{}_{\bar{\sigma}} \equiv \partial \lambda^{\bar{\gamma}\beta}{}_{\bar{\sigma}} A_\beta / \partial A_\alpha = \lambda^{\bar{\gamma}\alpha}{}_{\bar{\sigma}}$. This relation can be represented through an infinite set of coupled integral Dyson equations containing all dressed vertices. As argued in Refs. [23,29], however, vertex corrections give a finite and small modification and will be ignored. The dominant contributions in both the IR and the UV regions of the loop momentum integrals over instantaneous propagators come from diagrams with a maximal number of soft Coulomb lines and a maximal number of *primitive* self-energy loops, respectively. The *primitive* self-energy is shown in Fig. 2 and is given by

$$I^{0\bar{\alpha}}{}_{\bar{\beta}} \equiv \sum_\gamma \frac{1}{2\omega_\gamma}[\lambda_\gamma \lambda^\gamma]^{\bar{\alpha}}{}_{\bar{\beta}} = \sum_\gamma \frac{1}{2\omega_\gamma} \lambda^{\bar{\alpha}}{}_{\gamma\bar{\sigma}} \lambda^{\bar{\sigma}\gamma}{}_{\bar{\beta}} = \delta^{\bar{\alpha}}{}_{\bar{\beta}} I^0_{\alpha} \quad (33)$$

or

$$I^0_{\alpha} = I^0(q) = g^2 N_C \int [d\mathbf{k}] \frac{1-(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2}{2\omega(|\mathbf{k}|)(\mathbf{q}-\mathbf{k})^2}. \quad (34)$$

This self-energy is UV divergent and has to be renormalized. We will discuss renormalization in the following section. To proceed we need to introduce the expectation value of the inverse of the FP operator,

$$d^{\bar{\alpha}}{}_{\bar{\beta}} = \langle [(1-\lambda)^{-1}]^{\bar{\alpha}}{}_{\bar{\beta}} \mathcal{J}[A]\rangle / \langle \mathcal{J}[A]\rangle = \delta^{\bar{\alpha}}{}_{\bar{\beta}} d_{\bar{\alpha}}. \quad (35)$$

A few lowest order diagram contributions to this VEV are shown in Fig. 3. At the two-loop order the first and second diagram in the second line dominate in the IR and UV, respectively, and are retained. In higher orders the dominant contribution comes from a series of rainbow-ladder diagrams obtained by summing the class of diagrams generated by

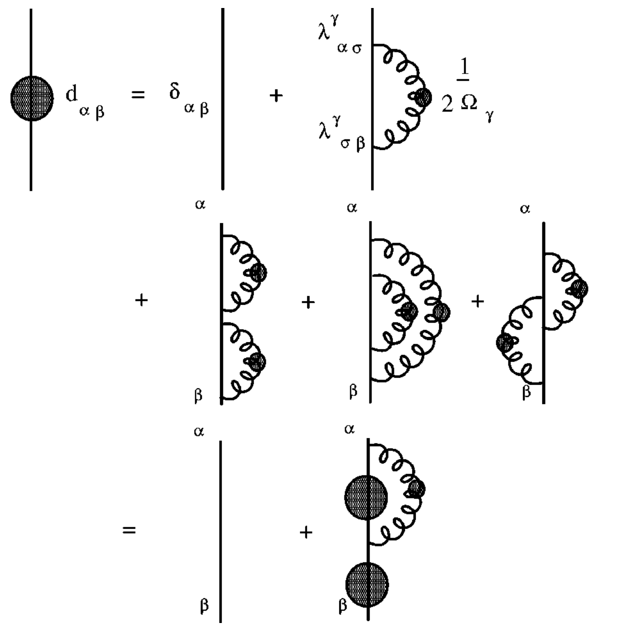

FIG. 3. Rainbow-ladder approximation to the Dyson equation for the Faddeev-Popov operator.

these two lowest order loop diagrams. The resulting approximation to the Dyson series is given by

$$d^{\bar\alpha}_{\bar\beta} = \delta^{\bar\alpha}_{\bar\beta} d_{\bar\alpha} = \delta^{\bar\alpha}_{\bar\beta} + \langle [\lambda(1-\lambda)]^{\bar\alpha}_{\bar\beta} \mathcal{J}\rangle / \langle \mathcal{J}\rangle = \delta^{\bar\alpha}_{\bar\beta} + \sum_\gamma \frac{1}{2\Omega_\gamma}[\lambda_\gamma d \lambda^\gamma d]^{\bar\alpha}_{\bar\beta} = \delta^{\bar\alpha}_{\bar\beta}[1 + I_{\bar\alpha} d_{\bar\alpha}], \tag{36}$$

where

$$\sum_\gamma \frac{1}{2\Omega_\gamma} \lambda^{\bar\alpha}_{\gamma\bar\sigma} d_{\bar\sigma} \lambda^{\bar\sigma\gamma}_{\bar\beta} = \delta^{\bar\alpha}_{\bar\beta} I_{\bar\alpha}. \tag{37}$$

With $d_{\bar\alpha} = d(q)$ and $I_{\bar\alpha} = I(q)$, we obtain the following equation:

$$d(q) = \frac{g}{1 - g I(q)},$$

$$I(q) = N_C \int [d\mathbf{k}] \frac{(1 - (\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2)}{2\Omega(|\mathbf{k}|)(\mathbf{q}-\mathbf{k})^2} d(|\mathbf{q}-\mathbf{k}|). \tag{38}$$

Using the same approximation (of ignoring vertex corrections), the Dyson equation for the instantaneous propagator becomes

$$\frac{\delta}{\delta J^\alpha}\frac{\delta}{\delta J_\beta}_{J=0} \left\langle \mathcal{J}\left[A + \frac{J}{2\omega}\right]\right\rangle \Big/ \langle \mathcal{J}[A]\rangle$$
$$= -\left[\frac{\lambda^\alpha}{2\omega_\alpha} d \frac{\lambda_\beta}{2\omega_\beta} d\right]^{\bar\gamma}_{\bar\gamma} + \sum_\rho \left[\frac{\lambda^\alpha}{2\omega_\alpha} d \lambda_\rho d\right]^{\bar\gamma}_{\bar\gamma} \frac{1}{2\Omega_\rho}\left[\lambda^\rho d \frac{\lambda_\beta}{2\omega_\beta} d\right]^{\bar\sigma}_{\bar\sigma}, \tag{39}$$

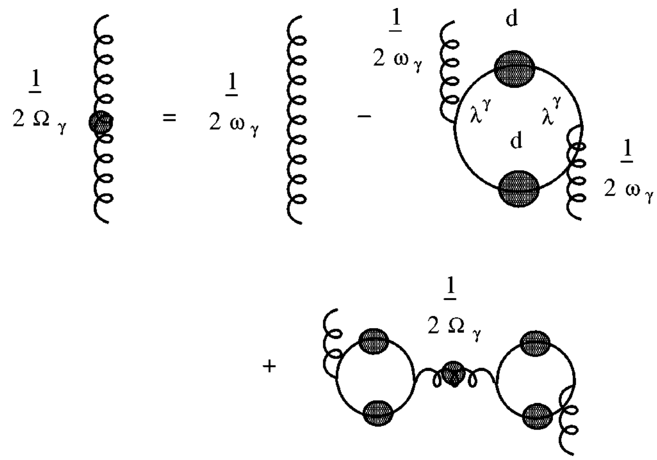

FIG. 4. The rainbow-ladder approximation to the Dyson equation for the transverse gluon propagator.

and is shown in Fig. 4. Since neutrality of the vacuum implies

$$[\lambda^\alpha d \lambda_\beta d]^{\bar\gamma}_{\bar\gamma} = \sum_{\gamma\sigma} \lambda^{\bar\gamma\alpha}_{\bar\sigma} d_{\bar\sigma} \lambda^{\bar\sigma}_{\beta\bar\gamma} d_{\bar\gamma} = 2\delta^\alpha_\beta F_\alpha, \tag{40}$$

we finally obtain

$$\Omega_\alpha = \omega_\alpha + F_\alpha, \tag{41}$$

where $F_\alpha = F(q)$ is given by

$$F(q) \equiv \frac{N_C}{2}\int [d\mathbf{k}] \frac{1 - (\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2}{(\mathbf{q}-\mathbf{k})^2} d(|\mathbf{k}|) d(|\mathbf{q}-\mathbf{k}|). \tag{42}$$

We can now return to the calculation of the vacuum expectation of the full Hamiltonian. Minimizing energy with respect to ω determines the ground state (and ω), and from Eq. (41), the gluon propagator $1/2\Omega$. In terms of this propagator, the kinetic vacuum expectation value E_K is given by

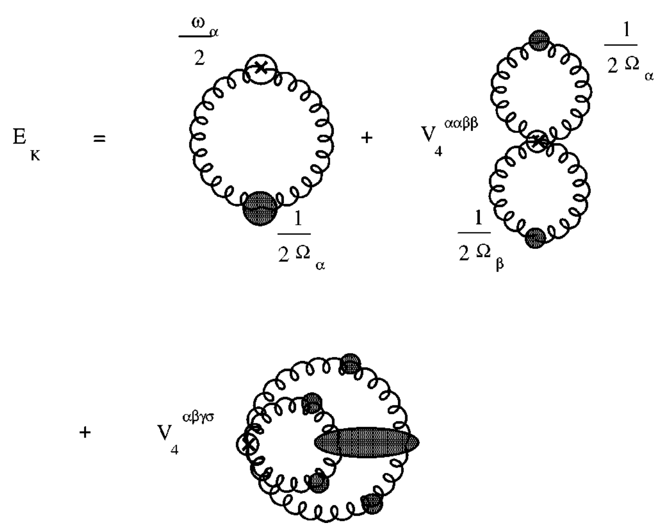

FIG. 5. The vacuum expectation value of the kinetic energy. The four-point function contribution comes from the $\mathbf{B}^2$ term.

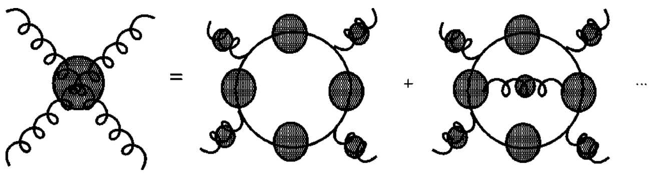

FIG. 6. Low order contributions to the 2PI gluon-gluon four-point function.

$$E_K=\frac{1}{2}\sum_{\alpha}(\omega_\alpha^2+p_\alpha^2)\Pi^{\alpha}{}_{\alpha}+\frac{1}{2}V_{\sigma\alpha\beta}V^{\sigma}{}_{\gamma\delta}\frac{\partial}{\partial J_\alpha}\frac{\partial}{\partial J_\beta}\frac{\partial}{\partial J_\gamma}\frac{\partial}{\partial J_\delta}\times e^{\sum_\rho (J^\rho J_\rho/4\omega_\rho)}\left\langle \mathcal{J}\left[A+\frac{J}{2\omega}\right]\right\rangle. \quad (43)$$

The second term originates from the square of the magnetic field, $B^{\sigma}=[\nabla\times]^{\sigma}{}_{\gamma}A^{\gamma}+V^{\sigma}{}_{\alpha\beta}A^{\alpha}A^{\beta}$, and it is shown in Fig. 5,

$$E_K/\mathcal{V}=\frac{1}{2}\int[d\mathbf{q}]\frac{\omega^2(q)+\mathbf{q}^2}{2\Omega(q)}+\frac{g^2N_C}{32}\int[d\mathbf{q}][d\mathbf{k}]\frac{3-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2}{\Omega(|\mathbf{k}|)\Omega(|\mathbf{q}|)}. \quad (44)$$

This magnetic contribution involves a transverse gluon four-point function, which as discussed earlier, is approximated by the product of two two-point functions, i.e. the gluon-gluon scattering amplitude shown in Figs. 6 is set to zero. The Coulomb energy VEV is shown in Fig. 7 and is given by

$$E_C=\frac{1}{2}\frac{\partial}{\partial J_\alpha}\frac{\partial}{\partial J_\beta}\frac{\partial}{\partial J_\gamma}\frac{\partial}{\partial J_\delta}\left[e^{\sum_\rho (J^\rho J_\rho/4\omega_\rho)}\rho_{\alpha\beta}\times\left\langle(1-\lambda)^{-2}(-\boldsymbol{\nabla}^2)\mathcal{J}\left[A+\frac{J}{2\omega}\right]\right\rangle\rho_{\gamma\delta}\right] \quad (45)$$

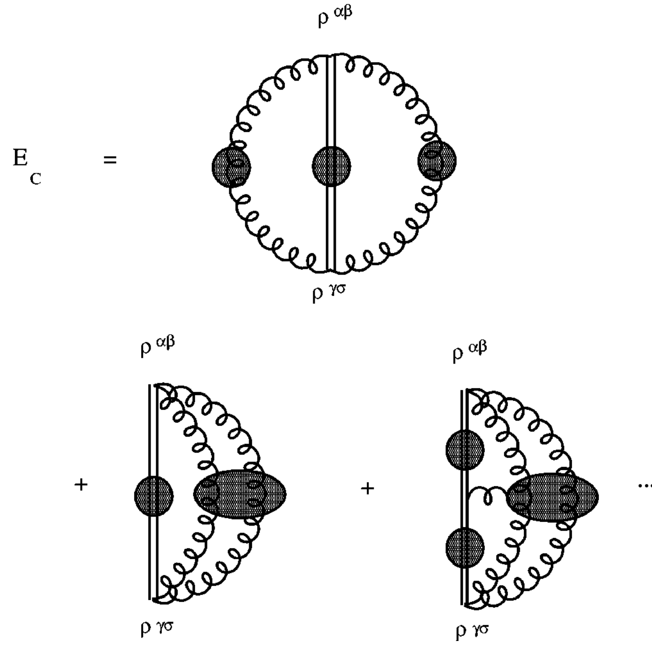

FIG. 7. The vacuum expectation value of the Coulomb operator.

with the charge density given by Eq. (22), $\rho=\rho^{\bar{\gamma}}=\rho^{\bar{\gamma}}{}_{\alpha\beta}A^{\alpha}A^{\beta}$, and

$$E_C/\mathcal{V}=\frac{N_C}{32}\int[d\mathbf{k}][d\mathbf{q}]\frac{K(|\mathbf{q}-\mathbf{k}|)}{(\mathbf{q}-\mathbf{k})^2}[1+(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2]\times\frac{[\Omega(|\mathbf{k}|)-F(|\mathbf{k}|)-\Omega(|\mathbf{q}|)+F(|\mathbf{q}|)]^2}{\Omega(|\mathbf{k}|)\Omega(|\mathbf{q}|)}. \quad (46)$$

Finally the gap equation follows from $0=\partial(E_K+E_C)/\partial\omega_\alpha$ and is given by

$$\Omega^2(q)-F^2(q)-\mathbf{q}^2=\frac{N_Cg^2}{4}\int[d\mathbf{k}]\frac{3-(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2}{\Omega(k)}+\frac{N_C}{4}\int[d\mathbf{k}][1+(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2]\times\frac{K(\mathbf{q}-\mathbf{k})}{(\mathbf{q}-\mathbf{k})^2}\frac{[\Omega(k)-\Omega(q)-F(k)+F(q)][\Omega(q)+\Omega(k)-F(k)+F(q)]}{\Omega(k)}, \quad (47)$$

with

$$K(q)=f(q)d^2(q), \quad (48)$$

and f satisfying

$$f(q)=1+N_C\int[d\mathbf{k}]\frac{1-(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2}{2\Omega(|\mathbf{k}|)(\mathbf{q}-\mathbf{k})^2}f(|\mathbf{q}-\mathbf{k}|). \quad (49)$$

It is also instructive to analyze the single quasiparticle dispersion relation

$$E_\alpha\delta^{\alpha}_{\beta}=\langle\mathcal{J}A^{\alpha}HA_{\beta}\rangle/\langle\mathcal{J}\rangle. \quad (50)$$

The calculation is straightforward although more tedious due to the presence of up to three contractions corresponding the VEV of six field operators. The final result is

$$E(q)=\frac{1}{2\Omega(q)}\Bigg[\Omega^2+F^2(q)+\mathbf{q}^2+\frac{N_C g^2}{4}\int[d\mathbf{k}]\frac{3-(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2}{\Omega(k)}+\frac{N_C}{4}\int[d\mathbf{k}](1+(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2)\frac{K(\mathbf{q}-\mathbf{k})}{(\mathbf{q}-\mathbf{k})^2}\frac{[\Omega(k)-\Omega(q)-F(k)+F(q)][\Omega(q)+\Omega(k)-F(k)+F(q)]+2\Omega^2(q)}{\Omega(k)}\Bigg]. \tag{51}$$

After combining with the gap equation one obtains

$$E(q)=\Omega(q)\left[1+\frac{N_C}{4}\int[d\mathbf{k}][1+(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2]\frac{K(\mathbf{q}-\mathbf{k})}{\Omega(k)(\mathbf{q}-\mathbf{k})^2}\right]. \tag{52}$$

In the limit $F=0$, corresponding to $\mathcal{J}=1$, Eqs. (47) and (52) reduce to the ones derived in Ref. [23] with $\Omega=\omega$.

IV. RENORMALIZATION

So far we have been ignoring potential UV divergences. These divergences should be removed by renormalizing appropriate operators and the coupling constant g. It turns out that all four equations of interest, Eqs. (38), (41), (47), (49), require renormalization. These equations have to be regularized first. This can be done by cutting off the momentum integrals, $\int[d\mathbf{k}]\to\int^\Lambda[d\mathbf{k}]$. The physical, renormalized solutions, $d(q)$, $f(q)$, $\omega(q)$ and $\Omega(q)$ should be Λ independent. We will first discuss renormalization of the expectation value of the inverse of the FP operator, $d(q)$. Assuming that a renormalized solution for $\Omega(q)$ has been found, the equation for $d(k)$ is renormalized by adjusting the bare coupling, $g\to g(\Lambda)$, i.e. the renormalized VEV of the inverse of the FP operator, $d(k)$ will play the role of the running coupling. The Λ dependence of $g(\Lambda)$ is determined by the UV behavior of Eq. (38),

$$\frac{dg(\Lambda)}{d\Lambda}=-\frac{\beta}{(4\pi)^2}\frac{g^2(\Lambda)d(\Lambda)}{\Omega(\Lambda)}, \tag{53}$$

with $\beta=8N_C/3$. In this and all other renormalization group equations we keep only relevant and marginal contributions, i.e. no power corrections, $O(p^n/\Lambda^n)$ with $n>0$ are included since they do not require renormalization. Since $\beta>0$ and physical solutions require $d(k),\Omega(k)>0$ the solution of Eq. (53) vanishes in the limit $\Lambda\to\infty$. In the limit $k\to\infty$ the integral $I(k=\Lambda,\Lambda)$ given by Eq. (38), with the second argument referring to the upper limit of integration, is finite. Thus in leading logarithmic approximation, $d(\Lambda)\to g(\Lambda)$ as $\Lambda\to\infty$. Furthermore, from Eq. (41) it follows that for large q, $q\sim\Lambda$,

$$\frac{d\Omega(q)}{dq}\to\frac{d\omega(q)}{dq}+O(d^2(q))\to\frac{d\omega(q)}{dq}+O(g^2(\Lambda)). \tag{54}$$

Similarly from Eq. (47), to leading logarithmic approximation, we find $d\omega(q)/dq=1+O(g^2(\Lambda))$ for $q\sim\Lambda$. Thus finally,

$$\Lambda\frac{dg(\Lambda)}{d\Lambda}=-\frac{\beta}{(4\pi)^2}g^3(\Lambda)+O(g^5(\Lambda)), \tag{55}$$

which, ignoring the terms $O(g^5)$, has a solution given by

$$g(\Lambda)=\frac{g(\mu)}{\left(1+\frac{\beta}{(4\pi)^2}g^2(\mu)\log(\Lambda^2/\mu^2)\right)^{1/2}}. \tag{56}$$

The asymptotic behavior as $\Lambda\to\infty$ is therefore given by

$$g(\Lambda)=\frac{4\pi}{\beta^{1/2}\log^{1/2}(\Lambda^2)}. \tag{57}$$

The renormalized equation for $d(k)$ is completely specified once $g(\mu)$, the value of the coupling at an arbitrarily chosen renormalization scale μ, is fixed. It should be stressed, however, that this solution is valid only to within terms of the order of $1/\log^{3/2}(\Lambda^2/\mu^2)$. In practical applications we will be renormalizing at a low energy scale μ, for example, by fixing the string tension or the glueball mass and thus such corrections become unimportant as $\Lambda\to\infty$. For relevant operators, however, as we will see below, such logarithmic corrections are multiplied by positive powers of Λ and thus cannot be neglected.

For practical (numerical) applications we have found a different, momentum subtraction renormalization (MSR) scheme to be more practical. In this scheme the renormalized equation for $d(q)$ is obtained by subtracting Eq. (38) at $q=\mu$,

$$\frac{1}{d(q)}-\frac{1}{d(\mu)}=-\frac{\beta}{(4\pi)^2}\int_{-1}^{1}d(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})\int_0^\infty dkk^2\frac{3}{4}\frac{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2}{\Omega(k)}\frac{d(\mathbf{q}-\mathbf{k})}{(\mathbf{q}-\mathbf{k})^2}+(q\to\mu). \tag{58}$$

In this renormalization scheme the coupling constant is therefore given by

$$\frac{1}{g_{MSR}(\Lambda)}=\frac{1}{d(\mu)}+\frac{\beta}{(4\pi)^2}\int_{-1}^{1}d(\hat{\mathbf{k}}\cdot\hat{\boldsymbol{\mu}})\int_0^\Lambda dk k^2\frac{3}{4}\times\frac{1-(\hat{\mathbf{k}}\cdot\hat{\boldsymbol{\mu}})^2}{\Omega(k)}\frac{d(\boldsymbol{\mu}-\mathbf{k})}{(\boldsymbol{\mu}-\mathbf{k})^2}, \tag{59}$$

and $g(\Lambda)=g_{MSR}(\Lambda)$ to within corrections of the order $O(\mu/\Lambda)$, i.e. they agree asymptotically. We also find for $q\sim\Lambda\to\infty$

$$d(q)=g_{MSR}(\Lambda)\{1+O[g_{MSR}^2(\Lambda)\log(\Lambda^2/q^2)]\}, \tag{60}$$

as expected from that discussed above. From now on we will only use the subtracted equations and drop the MSR subscript.

We now proceed to discuss renormalization of the equation for $f(k)$. Physically $f(k)-1$ represents the relative difference between the VEV of the square of the inverse of the FP operator and the square of the VEV of that operator. Any UV divergent contribution to f should therefore be renormalized by renormalizing the operator $g^2/(1-\lambda)^2$, since the operator $g/(1-\lambda)$ has already been renormalized. This is done by multiplying the Coulomb operator by a renormalization constant, $Z_K(\Lambda)$, $K[A]\to Z_K(\Lambda)K[A]$. Using the renormalized Coulomb operator the equation Eq. (49) for $f(k)$ becomes

$$f(k)=Z_K(\Lambda)+\frac{\beta}{(4\pi)^2}\int_{-1}^{1}d(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})\int_0^\Lambda dk k^2\frac{3}{4}\frac{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2}{\Omega(k)}\times\frac{d^2(\mathbf{q}-\mathbf{k})f(\mathbf{q}-\mathbf{k})}{(\mathbf{q}-\mathbf{k})^2}, \tag{61}$$

and in the limit $\Lambda\to\infty$ one obtains

$$\Lambda\frac{dZ_K(\Lambda)}{d\Lambda}=-\frac{\beta}{(4\pi)^2}\frac{d^2(\Lambda)f(\Lambda)}{\Omega(\Lambda)}Z_K(\Lambda), \tag{62}$$

which in the leading logarithmic approximation has a solution given by

$$Z_K(\Lambda)=\frac{Z_K(\mu)}{\log^{1/2}(\Lambda^2/\mu^2)}. \tag{63}$$

Choosing a value for $Z(\mu)$ at some UV point fixes the renormalized equation for $f(k)$. As in the case of the FP determinant, we will employ the momentum subtraction renormalization scheme, which leads to

$$f(k)-f(\mu)=\frac{\beta}{(4\pi)^2}\int_{-1}^{1}d(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})\int_0^\Lambda dk k^2\frac{3}{4}\frac{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2}{\Omega(k)}\times\frac{d^2(\mathbf{q}-\mathbf{k})f(\mathbf{q}-\mathbf{k})}{(\mathbf{q}-\mathbf{k})^2}-(q\to\mu), \tag{64}$$

resulting in the MSR scheme in $Z_K(\Lambda)$ given by

$$Z_K(\Lambda)=f(\mu)-\frac{\beta}{(4\pi)^2}\int_{-1}^{1}d(\hat{\mathbf{k}}\cdot\hat{\boldsymbol{\mu}})\int_0^\Lambda dk k^2\frac{3}{4}\times\frac{1-(\hat{\mathbf{k}}\cdot\hat{\boldsymbol{\mu}})^2}{\Omega(k)}\frac{d^2(\boldsymbol{\mu}-\mathbf{k})f(\boldsymbol{\mu}-\mathbf{k})}{(\boldsymbol{\mu}-\mathbf{k})^2}. \tag{65}$$

As expected, for UV values of $k\sim\Lambda\to\infty$ we find

$$f(k)=Z_K(\Lambda)\{1+O[g^2(\Lambda)\log(\Lambda^2/k^2)]\}. \tag{66}$$

An extensive discussion on renormalization of d and f has already been given in Ref. [23]. It should be noted that for large k, if $d^2(k)/\Omega(k)<1/\log(k)^n$ with $n>1$ there is no renormalization for $f(k)$. Our analysis suggests that $n=1$, thus the unrenormalized equation for f has a sub-leading $\log[\log(k)]$. We suspect that this is an artifact of the rainbow-ladder truncation.

As long as one works with the leading logarithmic approximation and uses $\Omega(k)=\omega(k)=k$, there is no effect of the FP determinant on d or f. The inclusion of the FP determinant influences the low momentum behavior of d and f, but it also introduces a new divergent integral, $F(q)$ in Eq. (42). Since the origin of F is the FP determinant $\mathcal{J}$, it is the FP determinant that has to be renormalized in order to make $\Omega(k)$ finite. The renormalized FP determinant should by chosen as

$$\mathcal{J}\to\left[\mathcal{J}e^{\sum_\alpha \delta\omega_\alpha A^\alpha A_\alpha+\cdots}\right]_\Lambda. \tag{67}$$

Here $\cdots$ stands for higher powers of the field operators; however, within the Gaussian approximation we are working with only the quadratic term that needs to be retained. It can be easily verified that replacing $\mathcal{J}$ by Eq. (67) leads to the replacement

$$F(q)\to F(q,\Lambda)+\delta\omega(\Lambda), \tag{68}$$

where $F(q,\Lambda)$ stands for the integral in Eq. (42) with the upper limit set to Λ. The counterterm $\delta\omega(\Lambda)$ will be chosen to make $\Omega(q)$ UV finite. Since $F(q)$ has the mass dimension of one, in general one expects two counterterms will be needed, one proportional to Λ and the other to one power of the momentum. From the UV behavior of the integrand in Eq. (42) it follows, however, that only the first is needed and we obtain

$$\frac{d\,\delta\omega(\Lambda)}{d\Lambda}=-\frac{\beta}{(4\pi)^2}d^2(\Lambda), \tag{69}$$

whose solution is given by

$$\delta\omega(\Lambda)=\delta\omega(\mu)-\frac{\beta}{(4\pi)^2}\int_\mu^\Lambda dk d^2(k). \tag{70}$$

We note that corrections to the leading asymptotic behavior $d(k)\sim g(k)$ cannot be neglected here since for $F(q)$ they result in terms of $O(\Lambda)$. Thus it is necessary to keep $d(k)$

rather then g in the renormalized expression for F. As in the case of d and f in the following we will use the MSR scheme for $\Omega(q)$, which gives

$$\Omega(q)-\Omega(\mu)-\omega(q)+\omega(\mu) = \frac{\beta}{(4\pi)^2}\int_{-1}^{1} d(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})\int_0^\infty dk k^2 \frac{3}{4}\frac{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2}{(\mathbf{q}-\mathbf{k})^2} d(\mathbf{k})d(\mathbf{q}-\mathbf{k}) -(q\to\mu), \tag{71}$$

with the asymptotic behavior $k\sim\Lambda\to\infty$ given by

$$\Omega(k)=\omega(k)+\frac{\beta}{(4\pi)^2}g^2(\Lambda)\Lambda\{1+O[g^2(\Lambda)\log(\Lambda^2/k^2)]\}, \tag{72}$$

and $\delta\omega(\Lambda)$ in MSR given by

$$\delta\omega(\Lambda)=\Omega(\mu)-\omega(\mu) - \frac{\beta}{(4\pi)^2}\int_{-1}^{1} d(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})\int_0^\Lambda dk k^2 \frac{3}{4}\frac{1-(\hat{\mathbf{k}}\cdot\hat{\boldsymbol{\mu}})^2}{(\boldsymbol{\mu}-\mathbf{k})^2} \times d(\mathbf{k})d(\boldsymbol{\mu}-\mathbf{k}). \tag{73}$$

The gap equation is the one which cannot be renormalized in a simple way. This is due to inconsistencies in the approximation used. Specifically, the gap equation is derived by taking the functional derivative of the energy expectation value with respect to ω. In the second integral in Eq. (47) we have retained only the derivative of the gluon lines and not of the Coulomb kernel. The former leads to terms in the integrand proportional to F thus formally of $O(g^4)$. Similarly derivatives of the Coulomb operator $(d^2 f)$ lead to terms proportional to $d^4 f/\Omega^2$, i.e. also of $O(g^4)$. Thus if terms proportional to the difference $\Omega-\omega$ are kept in the numerator of the gap equation it would be necessary to include derivatives of the Coulomb kernel. Since all these $O(g^4)$ terms involve two-loop integrals in the following we will neglect them. The simplified gap equation then reads

$$\omega^2(q)=\mathbf{q}^2+2F(q)\omega(q)+\frac{N_C g^2}{4}\int [d\mathbf{k}]\frac{3-(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2}{\omega(k)} + \frac{N_C}{4}\int [d\mathbf{k}][1+(\hat{\mathbf{q}}\cdot\hat{\mathbf{k}})^2]\frac{K(\mathbf{q}-\mathbf{k})}{(\mathbf{q}-\mathbf{k})^2}\frac{\omega^2(k)-\omega^2(q)}{\omega(k)}. \tag{74}$$

The equation is identical to the one in Ref. [23] except for the term involving F in the rhs and all terms are of $O(g^2)$. This simplification is justifiable since our goal is to study the effect of the FP determinant on the low momentum properties and thus possible modifications of UV behavior are largely irrelevant.

In a covariant formulation the renormalized theory has the same operator structure as the bare one. This is not the case in the Hamiltonian approach. Renormalization introduces non-canonical operators. The strength of such operators is determined by the cutoff. The gap equation has a quadratic divergence which is to be renormalized by a gluon "mass" counterterm in the Hamiltonian,

$$\delta H(\Lambda)=\frac{1}{2}m^2(\Lambda)A^\alpha A_\alpha. \tag{75}$$

This is the only relevant operator, e.g. of dimension two. The constant $m^2(\Lambda)$ is fixed by requiring that the gap equation leads to a Λ-independent solution. Thus we insist that $\omega(k)$ is Λ independent and this guarantees that any divergence of an operator matrix element calculated with respect to the state $|\omega\rangle$ will be associated with the operator itself and not with the state. The counterterm $\delta H(\Lambda)$ contributes to the rhs of Eq. (74) with $m^2(\Lambda)$ and from the UV behavior of Eq. (74) we find

$$\frac{dm^2(\Lambda)}{d\Lambda}=-\frac{\beta}{(4\pi)^2}\left[2g^2(\Lambda)\frac{\Lambda^2}{\omega(\Lambda)}+d^2(\Lambda)f(\Lambda)\omega(\Lambda)\right], \tag{76}$$

whose solution is

$$m^2(\Lambda)-m^2(\mu)=-\frac{\beta}{(4\pi)^2}\int_\mu^\Lambda \frac{dk}{\omega(k)}[2g^2(\Lambda)k^2 + d^2(k)f(k)\omega^2(k)]. \tag{77}$$

In the MSR scheme the gap equation then becomes

$$\omega^2(q)-\omega^2(\mu)-q^2-F(q)\omega(q)+F(\mu)\omega(\mu)+\mu^2 = \frac{\beta}{(4\pi)^2}\int_{-1}^{1} d(\hat{\mathbf{k}}\cdot\mathbf{q})\int_0^\infty dk k^2[\omega^2(k)-\omega^2(q)] \times\frac{3}{8}\frac{1+(\hat{\mathbf{k}}\cdot\mathbf{q})^2}{\omega(\mathbf{k})}\frac{K(\mathbf{q}-\mathbf{k})}{(\mathbf{q}-\mathbf{k})^2}-(q\to\mu), \tag{78}$$

with the asymptotic behavior for $k\sim\Lambda\to\infty$ given by

$$\omega^2(k)=q^2\{1+O[g^2(\Lambda)\log(\Lambda^2/k^2)]\} + \frac{\beta}{(4\pi)^2}g^2(\Lambda)Z_K(\Lambda)\Lambda^2, \tag{79}$$

and the counterterm $m^2(\Lambda)$ in the MSR is given by

$$m^2(\Lambda)=\omega^2(\mu)+\mu^2-2\frac{\beta}{(4\pi)^2}g^2(\Lambda)\int_0^\Lambda dk k^2\frac{1}{\omega(k)} - \frac{\beta}{(4\pi)^2}\int_{-1}^{1} d(\hat{\mathbf{k}}\cdot\hat{\boldsymbol{\mu}})\int_0^\Lambda dk k^2[\omega^2(k)-\omega^2(\mu)]\frac{3}{8} \times\frac{(1+(\hat{\mathbf{k}}\cdot\hat{\boldsymbol{\mu}})^2)}{\omega(\mathbf{k})}\frac{K(\boldsymbol{\mu}-\mathbf{k})}{(\boldsymbol{\mu}-\mathbf{k})^2}. \tag{80}$$

The renormalized equations for the VEV of the inverse of the FP operator, the corrections to d^2 needed to obtain the Coulomb potential, the gluon propagator and the ground state wave function are given by Eqs. (58), (64), (71), and (78), respectively. These equations depend on four parameters, the renormalization constants, $d(\mu)$, $f(\mu)$, $\Omega(\mu)$ and $\omega(\mu)$. In the following section we will study the solutions of these equations and their physical interpretation.

V. RESULTS

As discussed above there are four constants that need to be fixed. This can be done, for example, by comparing the Coulomb potential in position space,

$$V_{eff}(\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \frac{d^2(\mathbf{k}) f(\mathbf{k})}{\mathbf{k}^2} \tag{81}$$

with the lattice, static quark-antiquark potential. This procedure was used in Ref. [23]. Unfortunately, the dependence of V_{eff} on the renormalization constants is complicated, thus fitting the lattice potential will not necessarily provide much physical insight. Furthermore, it has recently been shown that there are differences between the lattice Coulomb potential and the static potential [45]. This is to be expected. On the lattice the Coulomb potential corresponds to the expectation value of a part of the Hamiltonian while the static potential gives the full energy.

We will thus proceed by simplifying the integral equations derived in the previous section and imposing constraints on the renormalization constants. The main difference between the present analysis and what was done in Ref. [23] has to do with the inclusion of the Faddeev-Popov determinant. Our goal here is to investigate the role of the FP determinant which is most prominently seen in the difference between $1/2\Omega$ and $1/2\omega$: the gluon propagator in the presence and absence of the FP determinant, respectively. If the determinant is omitted, one has $\Omega(k)=\omega(k)$ and in this case the remaining three equations, for $d(q)$, Eq. (58), $f(q)$, Eq. (64) and $\omega(q)$, Eq. (78), were analyzed in Ref. [23]. These equations have solutions provided $\omega(k)$ is finite as $k\to 0$. If $\omega(k)\to 0$ then the equation for $d(k)$ will develop a pole at a finite, positive value of momentum and if $\omega(k)\to\infty$ as $k\to 0$ then for a confining potential, $K(k)\to 1/k^{\alpha}$, with $\alpha>2$ the gap equation has no solution. A renormalization condition at $\mu=0$, $\omega(0)=m_g$ was therefore imposed with m_g fixed by the Wilson loop string tension (even though, as discussed above it is now known that the two string tensions are different). A simplified set of equations can be obtained by making an angular approximation,

$$|\mathbf{q}-\mathbf{k}| \to q\,\theta(q-k) + k\,\theta(k-q) \tag{82}$$

where $q=|\mathbf{q}|$ and $k=|\mathbf{k}|$. In Ref. [23] it was shown that the angular approximation leads to results which are very close to the exact numerical solutions. We will thus follow this approximation here since it allows us to considerably simplify the numerical analysis. After the angular approximation the equation for $d(k)$ becomes

$$\begin{aligned}\frac{1}{d(q)} - \frac{1}{d(\mu)} &= -\frac{\beta}{(4\pi)^2}\int_0^q dk \frac{k^2}{q^2}\frac{d(q)}{\Omega(k)} + \frac{\beta}{(4\pi)^2}\int_\mu^q dk \frac{d(k)}{\Omega(k)} \\ &\quad + \frac{\beta}{(4\pi)^2}\int_0^\mu dk \frac{k^2}{\mu^2}\frac{d(\mu)}{\Omega(k)},\end{aligned} \tag{83}$$

the gluon propagator function Ω is given by

$$\begin{aligned}\Omega(q) &= \Omega(\mu) + \omega(q) - \omega(\mu) + \frac{\beta}{(4\pi)^2}\int_0^q dk \frac{k^2}{q^2} d(k) d(q) \\ &\quad + \frac{\beta}{(4\pi)^2}\int_q^\mu dk d^2(k) - \frac{\beta}{(4\pi)^2}\int_0^\mu dk \frac{k^2}{\mu^2} d(k) d(\mu),\end{aligned} \tag{84}$$

and the gap equation for $\omega(q)$,

$$\begin{aligned}&\omega^2(q) - \omega^2(\mu) - q^2 - F(q)\omega(q) + F(\mu)\omega(\mu) + \mu^2 \\ &= \frac{\beta}{(4\pi)^2}\int_0^q dk \frac{k^2}{q^2} K(q) \frac{\omega^2(k)-\omega^2(q)}{\omega(k)} \\ &\quad + \int_q^\infty dk K(k) \frac{\omega^2(k)-\omega^2(q)}{\omega(k)} - (q\to\mu).\end{aligned} \tag{85}$$

We note that $d(q)$,$\Omega(q)/\mu$ and $\omega(q)/\mu$ are renormalization group (μ) invariants. As discussed above if one ignores the FP determinant it is not possible to choose an arbitrary renormalization condition for $\omega(0)$. Furthermore, in this case there is a critical (maximum) value of $d(\mu)=d_c(\mu)=4\pi\sqrt{3\beta\omega(0)/5\mu}$ for which Eq. (83) has a solution. The appearance of such a critical coupling is an artifact of the rainbow-ladder truncation used in evaluation of the expectation value of the Hamiltonian. The origin of this critical coupling can be illustrated by considering the following integral: a schematic representation of the functional integral representing the VEV of the inverse of the FP operator,

$$I(g) = \int dx \mathcal{J}(x) \frac{e^{-\omega x^2}}{1-gx}. \tag{86}$$

Here x represents the gauge potential, $\mathcal{J}(x)\sim e^{log(1-x)}$ plays the role of the FP determinant, and $1/(1-gx)$ of the VEV of the inverse of the FP operator. If one sets $\mathcal{J}=1$ the integral becomes divergent at $x=1/g$ unless $g=0$. In the ladder approximation the integral is evaluated by expanding $1/(1-gx)$ in a power series in gx and integrating term by term, keeping only a subset of contributions. It effectively means the approximation $\langle(x^2)^n\rangle=\langle x^2\rangle^n$ with $\langle x^m\rangle=\sqrt{\omega/\pi}\int dx x^m \exp(-\omega x^2)$, which gives

$$I(g) = \frac{1}{1-g^2/2\omega} \tag{87}$$

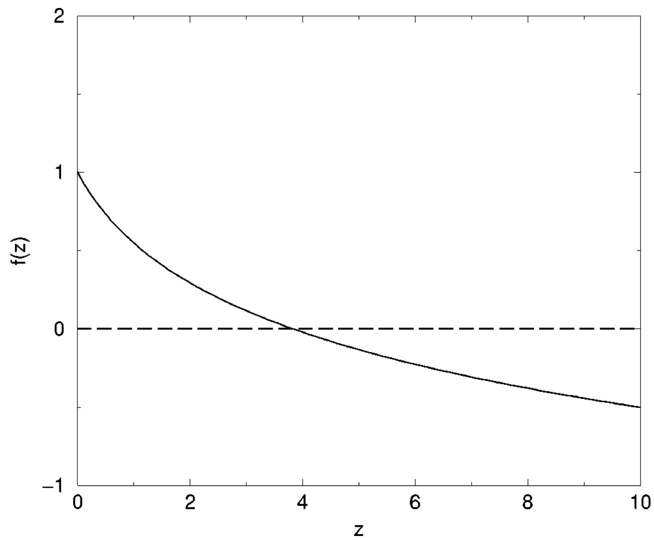

FIG. 8. A plot of the function $f(z)\equiv 1-[\text{rhs of Eq. (84)}]/\Omega(0) = -3\ln(1+5z/3)/5+31/25-25/125z+144(1-1/\sqrt{1+5z/3})/625z^2$, with $z\equiv\mu\beta d^2(0)/(4\pi)^2\Omega(0)$.

which has a critical coupling $g=g_c=\sqrt{2\omega}$.

The FP determinant, however, makes the integral well defined for all values of g and thus no critical coupling is expected if $\mathcal{J}$ is not ignored. This is indeed what happens if one uses Eq. (84). As long as $\Omega(0)>0$ the value of $\omega(q)$ at $q=0$ does not play a role in determining the position of the pole in the $d(p)$ and we can for simplicity assume $\omega(q)=0$. Then from Eq. (83) approximating $\Omega(q)=\Omega(0)$ for $q<\mu$, we obtain

$$d(q)=\frac{d(0)}{\left(1+\frac{5}{3}\frac{\beta}{(4\pi)^2}d^2(0)q/\Omega(0)\right)^{1/2}}. \tag{88}$$

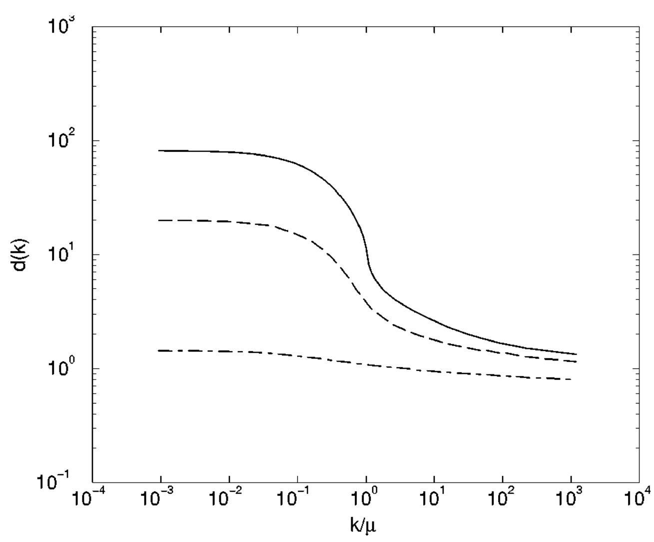

FIG. 9. Numerical solution for the vacuum expectation value of the inverse of the FP operator $d(k)$. The three curves correspond to $d(k=\mu)=38.4$ (solid), $d(k=\mu)=3.8$ (dashed) and $d(k=\mu)=1.1$ (dashed-dotted), respectively, indicating that a solution may exist for arbitrary choice of $d(\mu)$ i.e. no critical coupling.

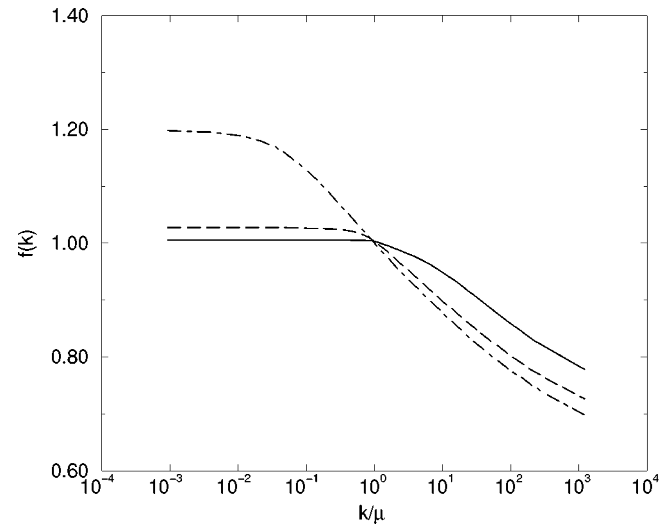

FIG. 10. Numerical solution for $f(k)$ normalized to $f(k=\mu)=1$. Labeling of curves is the same as in Fig. 9.

From Eq. (84) we can derive a relation between $\Omega(0)$ and $d(\mu)$,

$$z_0\Omega(0)=\mu\frac{\beta}{(4\pi)^2}d^2(0) \tag{89}$$

where $z_0\sim 4$ is a root of the nonlinear equation shown in Fig. 8. Now we see that as $d(0)$ increases so does $\Omega(0)/\mu$ but there is no upper limit on $d(0)$.

The FP determinant regularizes functional integrals near the Gribov horizon. As $\Omega(0)$ increases, the transverse-gluon two-point correlation function decreases at low momentum and the ghost correlator function $d(k)$ increases. This is pre-

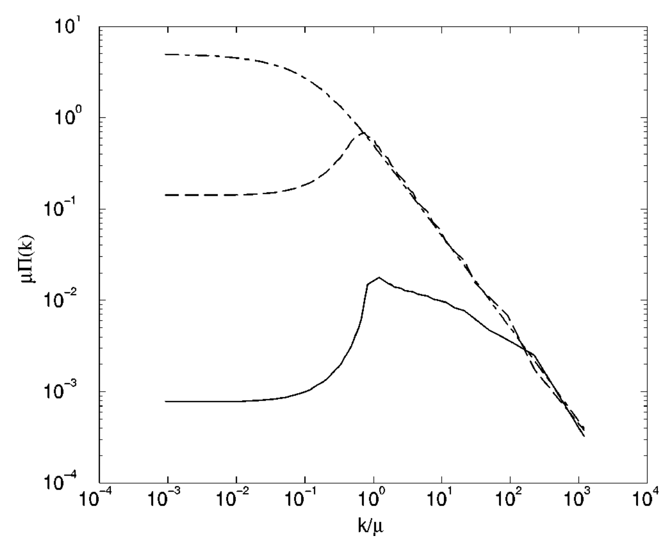

FIG. 11. Numerical solution for the instantaneous gluon propagator. Labeling of curves is the same as in Fig. 9. Increasing coupling $d(\mu)$ results in a stronger suppression at low moment.

cisely what was found in other gauges using Dyson-Schwinger methods and in other approximations to the Coulomb gauge.

In Figs. 9–11 we plot the results of numerical solutions to the set of coupled equations (64), (83), (84), (85) for $d(k)$, $f(k)$, and $\Pi(k)=1/2\Omega(k)$, respectively.

These should be compared with Figs. 4–6 from Ref. [23]. In Fig. 11 we plot the inverse of $\Omega(k)$ which is representing the transverse-gluon two-point function. As expected it is suppressed at low momenta and approaches the perturbative limit as $k\to\infty$.

We also note that even though the variational approach alone does not reproduce the correct UV limit of full QCD since for that propagating transverse gluons and quark-antiquark pairs are needed, these are not expected to change the IR behavior. The reason being that, as indicated by the leading contribution to the β function, the contribution from the propagating physical degrees of freedom is small, $O(10\%)$, and screening is thus not related to confinement. Furthermore, at low momenta the mass gap between the ground state and quark-antiquark and/or gluon states is of the order of the ρ-meson mass or the lightest glueball mass, and is not expected to mix in significantly. To some extent the decoupling of the IR and UV limits is seen from Eq. (88). The IR exponent of 1/2 is independent of the value of β.

VI. SUMMARY

In this paper we have studied the role of the Faddeev-Popov determinant in the Coulomb gauge. The FP determinant specifies the measure in the functional integrals over gauge field configurations and has so far been ignored in most calculations of QCD matrix elements in the Coulomb gauge. The FP determinant vanishes at the boundary of the Gribov region; nevertheless it still allows for large field configurations near the boundary to enhance matrix elements involving the inverse of the FP operator or the Coulomb potential. In particular we have shown that the inverse of the FP operator, corresponding to the running coupling or the ghost propagator, is strongly enhanced in the IR, but at the same time no artificial critical coupling exists. The same is true for the Coulomb kernel which is related to confinement. Finally, the instantaneous part of the transverse gluon propagator is found to be suppressed as is found in other gauges.

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